Can local caution restore global tacit collusion?: Repeated multimarket contact with observation errors

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Summary

- We analyze equilibria in repeated multimarket contact under private monitoring.
  - Multimarket contact: Two firms (players) operate in multiple markets (games) simultaneously.
  - Private monitoring: Each player may observe a noisy, different observation about the opponent’s action.
- 1-period mutual punishment (1MP) strategy
  - Necessary and sufficient equilibrium conditions
- Locally-cautioning (LC) strategy
  - We numerically identify the parameters for the equilibria
  - For two market case, LC can achieve 98% of efficiency, while a naive extension of 1MP does 87% at maximum.
Outline

• Repeated games
• Repeated multimarket contact with private monitoring
• Strategy representation and equilibria
  – Belief-based strategy
  – 1-period mutual punishment strategy
  – Locally-cautioning strategy
• Conclusion

Prisoners’ dilemma

• “Defect” is strictly dominant, while higher payoffs for both players are achieved if both “Cooperate.”
• In an isolated interaction, there is no chance to escape from this “dilemma.”
• We must accept the inefficient outcome if we believe players are rational.

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<tbody>
<tr>
<td>C</td>
<td>1, 1</td>
<td>-1, 2</td>
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<tr>
<td>D</td>
<td>2, -1</td>
<td>0, 0</td>
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Repeated games

• However, in real life, people seem to achieve cooperation quite frequently in PD like situations.
• Why?
  – Because people are irrational?
  – Are people playing a different game?
• One convincing (arguably, only one widely accepted) explanation:
  – Rational players can cooperate in a long-term relation

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-> Infinitely repeated games

Model of repeated games

• The game (called the stage game) is played in periods 0, 1, 2, ....
• The players make their choices simultaneously in each period, and then observe that period’s outcome before proceeding to the next.
• Let \( a_i(t) \) be the action of player \( i \) in period \( t \),
  \( g_i(a_i(t), a_j(t)) \) be the payoff of player \( i \) for one stage game.
• Player \( i \) tries to maximize the normalized discount sum of payoffs defined below, where \( \delta \in [0, 1) \).

\[
(1 - \delta) \sum_{t=0}^{\infty} \delta^t g_i (a_i (t), a_j (t))
\]
• By this definition, if the same stage game payoff 1 is achieved, the normalized payoff becomes 1.
Perfect monitoring

• players can perfectly observe each other’s actions in the past.
• This case has been extensively studied; there exist rich theoretical results:
  – Folk theorem: any cooperative outcome is achieved in an equilibrium.
  – One-shot deviation principle: checking whether a strategy profile constitutes an equilibrium, where each strategy is described as a Finite State Automaton (FSA), is easy.

Imperfect monitoring

• In reality, long term relationships are often plagued by imperfect monitoring.
  – players cannot directly observe each other's actions.
  – They observe signals that imperfectly reveal what actions have been taken.
• Repeated games with imperfect monitoring are classified into two categories:
  – Public monitoring: players commonly observe a public signal.
  – Private monitoring: each player observes a signal that is not observable to others.
## Public monitoring

<table>
<thead>
<tr>
<th></th>
<th>$w_2 = g$</th>
<th>$w_2 = b$</th>
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<tbody>
<tr>
<td>$w_1 = g$</td>
<td>$p$</td>
<td>0</td>
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<tr>
<td>$w_1 = b$</td>
<td>0</td>
<td>$s$</td>
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**When $(a_1, a_2) = (C, C)$**

Signal $(g, g)$ is correct

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<thead>
<tr>
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<th>$w_2 = b$</th>
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</thead>
<tbody>
<tr>
<td>$w_1 = g$</td>
<td>$p'$</td>
<td>0</td>
</tr>
<tr>
<td>$w_1 = b$</td>
<td>0</td>
<td>$s'$</td>
</tr>
</tbody>
</table>

**When $(a_1, a_2) = (C, D)$**

Signal $(b, g)$ is correct

---

**No error ($p$) > Two error ($s$)**

**One error ($p' < p$) < One error ($s'$)**

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  - Locally-cautioning strategy
- Conclusion
Private monitoring
(nearly-perfect monitoring)

<table>
<thead>
<tr>
<th>When ((a_1, a_2) = (C, C))</th>
<th>When ((a_1, a_2) = (C, D))</th>
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</thead>
<tbody>
<tr>
<td>Signal ((g, g)) is correct</td>
<td>Signal ((b, g)) is correct</td>
</tr>
<tr>
<td>(w_1 = g) (w_2 = g)</td>
<td>(w_1 = g) (w_2 = b)</td>
</tr>
<tr>
<td>(w_1 = b) (w_2 = q)</td>
<td>(w_1 = b) (w_2 = q)</td>
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</table>

No error \((p)\) > Two error \((s)\) > One error \((q)\)

Multimarket contact

- Two players operate in multiple markets (prisoners’ dilemma game) simultaneously (Bernheim and Whinston 1990).
  - A company sells a variety of goods across multiple markets and often supplies an identical good to areas that are geographically apart, e.g., U.S. and Japan.
  - Tacit collusion among companies is likely to occur. However, little is theoretically known in the private monitoring case.
- Public monitoring (Kobayashi and Ohta 2012)
  - A sufficient condition that a generalization of the trigger strategy profile is an equilibrium with collusive behavior (attains the highest average payoffs).
- Private monitoring (This work)
  - The generalized trigger strategy is no longer an equilibrium.
  - An extension of 1MP strategy attains approximately 87% of cooperative level.
  - We found a “locally-cautioning” strategy that attains approximately 98%.
Base strategy: 1-period mutual punishment (1MP)

- A player first cooperates (state R).
- When she receives a bad signal, she defects at the next period (state P).
- At that period, she again observes a bad signal, she returns to cooperate.
- J-1MP: A player acts according to 1MP in each market
  - If 1MP is an equilibrium in one market, it is in an arbitrary number of markets.

New strategy: Locally-cautioning (LC)

- A player first cooperates in two markets (state R).
- When she receives bad signals in both markets, she defects in a single market at the next period (state P).
- At that period, if she observes a good signal in the market she keeps to cooperate, she returns to cooperate.
- This strategy attains much higher payoffs than J-1MP, though it is not so robust as J-1MP against observation errors.

*: g or b
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Strategy of repeated games

• To verify an equilibrium, we require a strategy that maps any history of past actions and past observed signals to a current action.
• In private monitoring, because the opponent’s past actions and signals are not observable, the number of possible histories exponentially grows.

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
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<tbody>
<tr>
<td>Player 1</td>
<td>action</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>signal</td>
<td>g</td>
<td>g</td>
<td>b</td>
</tr>
<tr>
<td>Player 2</td>
<td>signal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>action</td>
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Belief-based strategy (Kandori and Obara 2010)

- Represents the opponent's behavior as a finite state automaton (FSA).
- Player 1 has a belief over states of Player 2.
- If the number of states is two, her belief is the probability that Player 2 is in state R.
- A strategy is compactly represented as a mapping from a current belief to a current action.

\[ b_1 = \Pr(\theta_2 = R) = b \]

Player 1

Value function determines a current action

- Given a setting and a FSA profile, obtain \( v^{(i,j)} \) by solving Markov equations.
- Belief-based continuation payoff when Player 1 acts based on FSA \( m_i \) starting from R:
  \[ V_i(m_i, R)(b_i) = b_i v^{RR} + (1 - b_i) v^{RP} \]
  \[ V_i(m_i, P)(b_i) = b_i v^{PR} + (1 - b_i) v^{PP} \]
- Belief-based value function:
  \[ V_i(m_i)(b_i) = \max\{V_i(m_i, R)(b_i), V_i(m_i, P)(b_i)\} \]
- A belief is updated by Bayes’ rule from the action and observation.

The theory of POMDP (dynamic programming) enables us to verify whether a strategy (policy) profile constitutes a sequential equilibrium!
Full characterization of 1MP

• Checking all one-shot extensions (backup operator in POMDP), we obtain the following characterization.

• Under nearly-perfect monitoring with $p, q, s=1-p-2q$ ($p>s>q$), 1MP starting from R constitutes an equilibrium if and only if

$$\delta \geq \frac{(p-s+X)x + (p-s-X)y}{2(p-s)X}$$  \hspace{1cm} \text{(Eq. 1)}$$

$$\delta \geq \frac{Y + \sqrt{Y^2 + 4(p-s)(s-q)x}}{2(p-s)(s-q)}$$  \hspace{1cm} \text{(Eq. 2)}$$

• Note that

$$X = p + q - \sqrt{(q-s)^2 + 4pq}$$

$$Y = -p + qy + (1+x)s$$

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Let us plot the conditions

\[ \delta=0.9, \ x=y=0.12 \]
Locally-cautioning (LC) revisited

- A player first cooperates in two markets (state R).
- When she receives bad signals in both markets, she defects in a single market at the next period (state P).
- At that period, if she observes a good signal in the market she keeps to cooperate, she returns to cooperate.
- It is very difficult to analytically verify this strategy
  - \(4^2 = 64\) policies must be checked one by one.

\begin{align*}
& (g,*) , (b,g) \\
& (b,*), (g,b) \\
\end{align*}

Let us verify it numerically

\( a_i = (C,C) \quad a_i = (C,D) \)

*: g or b

Region that LC constitutes an equilibrium in two market case

\begin{align*}
& s = q \\
& p + 2q = 1 \\
\end{align*}

\( \delta = 0.9, x = y = 0.12 \)

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**Why LC works?**

- LC is never an equilibrium under perfect monitoring.
  - At state R, even if you defect in a single market, you are never punished.
- Under imperfect monitoring, at state R, when you defect, you is more likely to be punished.
  - The current gain from deviation can exceed the future loss caused by that deviation.
  - After the opponent’s deviation, though you punish (caution) him only in Market B, your opponent is likely to realize which state you are at.
  - Thus, you and your opponent can quickly return to cooperation simultaneously.
- We can construct a strategy such that you punish in both markets (globally-cautioning, GC).
  However, the average payoff becomes small.
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Conclusion

• Analyze equilibria in repeated multimarket contact under private monitoring.
  – Derive an equilibrium condition of 1MP.
  – Numerically find a new strategy, LC.
• Future work
  – Theoretical analysis of LC (belief-free equilibrium?)
  – Explore a equilibrium strategy in more than three markets.

δ=0.9, x=y=0.12, q=0.04