Designing Fair, Efficient, and Incentive Compatible Team Formation Markets

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Suppose you need to divide agents into teams...

- Classroom groups
- Research teams
- Disaster relief teams
...subject to natural constraints:

- Agent preferences over teams must be elicited
- Team sizes must fall in a given range
- Agents may not exchange money
Our contributions

1) Introduce the *team formation problem*

2) Propose 2 novel mechanisms:
   a) *One-player-one-pick draft*
   b) *Approximate competitive equilibrium from equal incomes* (for team formation)

3) Prove basic properties of our mechanisms

4) Empirically analyze mechanism outcomes on real and synthetic data
Talk outline

• Prior work: *Hedonic games*
• Proposed mechanisms
  – Background: *Competitive equilibrium from equal incomes*
• Experiments
• Results
• Conclusion
PRIOR WORK
Hedonic games are a type of coalition formation game

\[(N, \succ) \rightarrow P\]

- Agents \(N\) must be partitioned into teams \(P\)
- Each agent’s preferences \(\succ\) are based only on its team, not the other coalitions
Negative results for hedonic games

• Core maybe be empty, even for additive separable preferences (Banerjee 2001)
• Testing core emptiness is NP-complete, even for additive separable preferences (Sung 2010)
Team formation problem

\((N, \succ, \underline{k}, \bar{k})\)

- Team sizes must be within \([\underline{k}, \bar{k}]\)
- Agent preferences, \(\succ\), must be elicited by the mechanism
- We assume \textit{additive separable} preferences
  - Our data sets list utility of individuals, not of teams
  - Natural way to induce utility of a set from individuals
Desirable properties for a team formation mechanism

• Efficiency
  – High social welfare
  – \textit{Ex post} Pareto efficiency

• Fairness
  – Lack of envy

• Truthfulness
  – Strategyproofness
  – Low \textit{regret of truthful reporting}
We cannot bound envy by a single teammate in team formation

- **Envy-freeness of a partition:**
  - No agent would prefer to switch teams with another agent

- **Envy bounded by a single teammate:**
  - For any agent A that would prefer to switch teams with B:
    - There exists another agent C teamed with B,
    - Where removing C makes A prefer its original team
\[ (k = 3, \bar{k} = 3) \]
PROPOSED MECHANISMS
Random serial dictatorship

1) Agents are randomly ordered
   a) First $numTeams$ agents are “captains”
2) Next captain in order selects entire team
1) Agents are randomly ordered
   a) First \textit{numTeams} agents are “captains”

2) Next captain in order selects 1 teammate
   a) Order of captains is reversed in next iteration
One-player-one-pick draft

1) Agents are randomly ordered
   a) First \(numTeams\) agents are “captains”

2) Iterate once over agents
   a) If not on a team: Join most-preferred team with a vacancy
   b) If on a team with a vacancy: Select favorite free agent to recruit
One-player-one-pick draft

If not on a team:
One-player-one-pick draft

Else if on a team with a vacancy:
Background

COMPETITIVE EQUILIBRIUM FROM EQUAL INCOMES
Multi-unit allocation problem

\[(N, M, q_i^M, \Psi_j^N, \succeq) \rightarrow x\]

- Items \(M\) must be assigned to agents \(N\), based on preferences \(\succeq\)
- Each item \(i\) has a quantity \(q_i\)
- Each agent \(j\) can accept sets \(\Psi_j\)
- Result is an allocation \(x\) to the agents
Competitive equilibrium from equal incomes

1) Agents report preferences over item sets
2) Assign each agent an equal budget of “fake money”
3) Find prices such that supply equals demand for each good
4) Assign goods to agents based on demands at these prices
CEEI Example

Supply: \{1 , 1 \}

A: (2 = 1 ), budget = $1, capacity = 1
B: (4 = 1 ), budget = $1, capacity = 1

price = ( : $2/5, : $8/5)

\[ x = \{A:(\frac{1}{2} , \frac{1}{2} ), B:(\frac{1}{2} , \frac{1}{2} )\} \]
Problem:

With indivisible goods, market-clearing prices for CEEI may not exist.
Indivisible goods example  (Budish 2011)

Supply: (1 ♦️, 1 ♦️, 1 🌈, 1 ☔️️)

A: budget = $1, capacity = 2
B: budget = $1, capacity = 2

For any prices, A and B have equal demands. This market cannot clear with equal budgets.
Approximate competitive equilibrium from equal incomes

• Let each agent have a budget, $b$, in:

$\left(1, 1 + \frac{1}{k-1}\right)$

• Find prices that approximately clear the market.

• Guarantees envy bounded by a single good, and approximate market clearing. (Budish 2011)
A-CEEI example (Budish 2011)

Supply: (1 diamond, 1 square, 1 blue crystal, 1 rock)

A: budget = $1.2, capacity = 2
B: budget = $1, capacity = 2

price = (1.1, 0.8, 0.2, 0.1)

x = {A:(1 diamond, 1 square), B:(1 diamond, 1 square)}
EXPERIMENTS
## Data Sets

|                      | $C$  | $|N|$ | $k$ | $\bar{k}$ |
|----------------------|------|------|-----|----------|
| Random-similar 20    | 0.914| 20   | 5   | 5        |
| Random-scattered 20  | 0.499| 20   | 5   | 5        |
| Newfrat              | 0.877| 17   | 4   | 5        |
| Freeman              | 0.551| 32   | 5   | 6        |
Key difference among data sets: Similarity of agent preferences

- If agents’ preferences are similar, there is less distinction between the mechanisms.
- Measure of preference similarity: mean pairwise cosine similarity

\[
C = \frac{\sum_{i=1}^{|N|} \sum_{j=i+1}^{N} \frac{u_{i-j} \cdot u_{j-i}}{||u_{i-j}|| ||u_{j-i}||}}{\left(|N|^2 - |N|\right)/2}
\]
## Data Sets

| Data Set                  | $C$    | $|N|$  | $k$ | $\bar{k}$ |
|--------------------------|--------|-------|-----|----------|
| Random-similar 20        | 0.914  | 20    | 5   | 5        |
| Random-scattered 20      | 0.499  | 20    | 5   | 5        |
| Newfrat                  | 0.877  | 17    | 4   | 5        |
| Freeman                  | 0.551  | 32    | 5   | 6        |
Brute-force search over partitions is intractable for these data sets

\[
\frac{17!}{(4!)^3(5!)(4!)} = 8933925 \quad \text{Newfrat}
\]

\[
\frac{20!}{(5!)^4(4!)} = 488864376 \quad \text{Random-20}
\]

\[
\frac{32!}{(6!)^2(5!)^4(6!)} \approx 3.4 \times 10^{18} \quad \text{Freeman}
\]
Estimating regret of truthful reporting

1) Generate $j$ random deviations $D$ per agent, from the agent’s true preferences
   a) Record each agent’s change in payoff when the agent reports $D$ and all others report truthfully

2) Repeat $k$ times for each data set

3) Result is the mean over $k$ runs, of the greatest gain achieved by any agent
RESULTS
OPOP empirically yields better social welfare than RSD, HBS, A-CEEI*

* For data sets with “dissimilar” preferences
OPOP empirically yields lower regret of truthful reporting than A-CEEI, Max-welfare
OPOP empirically *bounds envy by a single teammate* for more agents than RSD, HBS, A-CEEI*

* HBS is close to OPOP when agents’ preferences are highly similar
OPOP is more fair over serial order than RSD, HBS, A-CEEI
OPOP and HBS are more fair to unpopular agents than RSD
Conclusions

- *OPOP draft* empirically yields good truthfulness, fairness, and social welfare.
- *A-CEEI* is empirically worse than *OPOP* in fairness and truthfulness.
  - Even though it is strategyproof-in-the-large.
- *RSD* is the least fair of these mechanisms.
  - But it is ex post Pareto efficient, strategyproof.
Questions?
References