Commitment versus Flexibility in Law Enforcement Games

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Enforcement games

- General Setting

Examples:
- An employee chooses whether or not to work; an employer chooses how much to invest in monitoring the worker.
- A taxpayer chooses whether or not to report income truthfully; a tax authority chooses how frequently to audit taxpayers.
- A criminal chooses whether or not to commit an offense; the police choose how much to invest in prevention or detection of crime.
- A state chooses whether or not to develop a nuclear facility; a UN's nuclear inspection team decides on frequency of inspection.
Enforcement games

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Law Enforcement Games

- Stackelberg game
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  - Enforcer commits to an observable investigation strategy
Stackelberg game

- Enforcer commits to an observable investigation strategy
  - Investigation = detection of offender conditional on non-compliance
- Detection of non-compliance does not rectify the harm from non-compliance
- Examples: Speeding or parking violation, bodily-injury crimes
- Subgame perfect Nash equilibrium
  - Given enforcement strategy, offender's choice of non-compliance is a best response
- Offender cannot threaten to never comply
- Enforcer's strategy is supported by commitment power (not subgame perfect)
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Inspection Games

- Simultaneous game
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  - Enforcer and offender choose strategies independently

Monitoring = detection of non-compliance

Enforcement is aimed at preventing non-compliance

Detection of non-compliance rectifies (at least partially) the harm from non-compliance

Example: Inspection of a nuclear facility

Mixed-strategy equilibrium

Since strategy spaces contain two actions, each player must be indifferent between his actions

Sanction doesn't affect crime, but only enforcement level (Tsebelis, 1989)
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The Value of Commitment in Enforcement Games

The Criminal Investigation Division of the Environmental Protection Agency must choose whether

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announce, and commit to, a uniform investigation policy of a class of property crimes such as thefts and burglaries involving more than $100,000; or

to invest in investigating and detecting offenders involved in such crimes after having observed the gravity of the crime as measured, for example, by the amount stolen.
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Model highlights

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  - Offender’s commitment to non-compliance is structural (pollution, theft)
Main results

1. Enforcer enjoys a first-mover advantage

Enforcer's payoffs are higher as a leader than in a Nash game.

2. Enforcer enjoys a second-mover advantage

Enforcer's payoffs are higher as a follower than in a Nash game.

3. Enforcer's equilibrium payoffs may be higher as a follower than as a leader

Depending on the Enforcer's first-versus second-mover advantage.
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  - Strategies
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  - Payoffs

- Reaction curves and Nash equilibrium
- Enforcer-leadership game
- Offender-leadership game
- Comparison of games
- Conclusion
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**Strategies**

- Offender chooses level of non-compliance, $q \in [0, 1]$

  - Marginal gain from non-compliance is positive and decreasing:
    \[
    G_0(q) > 0, \quad G_{00}(q) < 0, \quad G_0(1) = 0
    \]

- Enforcer chooses probability of detection, $p \in [0, 1]$

  - Cost of detection is:
    \[
    c(p)
    \]
  
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- Detection rectifies (ex ante or ex post) a portion of the harm

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    - Property crime
Harm and sanction are proportional to non-compliance
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Sanction: \( qS \)
Payoffs

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- Sanction: $qS$
- Harm: $qH$
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  - Harm: \( qH \)
- Offender’s payoff:

\[
v(p, q) = (1 - p)G(q) - pqS.
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- Offender’s payoff:
  \[
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  \]

- Enforcer’s payoff:
  \[
  \mu(p, q) = -\left(1 - p\right)qH - c(p).
  \]
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Offender’s marginal gain from non-compliance is decreasing in level of enforcement ($\frac{dv}{dqdp} < 0$)
Offender’s Reaction Curve

- From the offender’s perspective, strategies are substitutes
  - Offender’s marginal gain from non-compliance is decreasing in level of enforcement \( \frac{dv}{dqdp} < 0 \)
  - Offender’s best response is decreasing in level of enforcement
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- Offender’s marginal gain from non-compliance is decreasing in level of enforcement \( \left( \frac{dv}{dq dp} < 0 \right) \)

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- Offender’s best-response payoff is increasing in level of non-compliance \( \left( \frac{dv}{dq} \bigg|_{p_{br}(q)} > 0 \right) \)
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- Offender’s best-response payoff is increasing in level of non-compliance \( \left( \frac{dv}{dq} \bigg|_{p_{br}(q)} > 0 \right) \)

  - That is, Offender’s payoff is decreasing along his best-response curve
From the enforcer’s perspective, strategies are complements
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- Enforcer’s marginal gain from enforcement is increasing in offender’s level of non-compliance \( \frac{du(p,q)}{dqdp} > 0 \)
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\(\text{Enforcer’s Reaction Curve}\)
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  - Enforcer’s marginal gain from enforcement is increasing in offender’s level of non-compliance \( \left( \frac{du(p,q)}{dqdp} > 0 \right) \)
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Iso Payoff Curves

Enforcer's Iso-Payoff Curves

Offender's Iso-Payoff Curves

- Payoff increases
- Zero profit line
Reaction Curves and Nash Equilibrium

Best Response Curves

- Enforcer's Reaction Curve
- Offender's Reaction Curve

$p$ and $q$ axes with values from 0 to 1.
Game of Conflict

- Each player’s best-response payoff decreases along his best-response curve
Enforcer-leadership game

- Time structure
Enforcer-leadership game

- Time structure
  - Stage 1: Enforcer commits to an observable level of enforcement
Enforcer-leadership game

Time structure

- Stage 1: Enforcer commits to an observable level of enforcement
- Stage 2: Offender chooses a level of non-compliance
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- **Time structure**
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- Enforcer chooses a point on Offender’s reaction curve to maximize its payoff
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- Enforcer’s optimal level of enforcement is higher relative to a simultaneous game
Enforcement not Lower in an Enforcer-Leadership than in a Nash Game

- Suppose Enforcer choose \( p^A \) (less enforcement relative to Nash); offender replies with \( q^A \)
Enforcement not Lower in an Enforcer-Leadership than in a Nash Game

- Suppose Enforcer choose $p^A$ (less enforcement relative to Nash); offender replies with $q^A$.
- Enforcer’s payoff is higher at B than at A ($p^B$ is a best response) and is higher at N than at B (Enforcer’s payoff is decreasing along his reaction curve) $\Rightarrow$ Enforcer should choose $p^N$ rather than $p^A$. 

![Enforcement Diagram](image-url)
Enforcer-Leadership Game Equilibrium

The diagram illustrates the Enforcer-Leadership Equilibrium in a two-dimensional graph. The axes are labeled with $q$ and $p$, indicating the variables in the game. The equilibrium is depicted by a point $(q^*, p^*)$ where the two curves intersect.
Enforcer’s Problem as a Leader

- Enforcer minimizes the expected harm from non-compliance plus enforcement cost
Enforcer’s Problem as a Leader

- Enforcer minimizes the expected harm from non-compliance plus enforcement cost
- Enforcer’s problem:

\[ \min_p [(1 - p) q_{br}(p) H + c(p)] \]
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- First Order Condition:

\[
\frac{\partial c(p)}{\partial p} - q H = \frac{d q_{br}(p)}{d p} (1 - p) H = 0
\]

- Direct Effect

\[c(p) - q H\]

Marginal net gain, given q

- Strategic Effect

\[dq_{br}(p)\]

Marginal deterrence gain
Offender-Leadership Game

- Time structure
Offender-Leadership Game

- Time structure
  - Stage 1: Offender chooses an observable level of non-compliance
Offender-Leadership Game

Time structure

- Stage 1: Offender chooses an observable level of non-compliance
- Stage 2: Enforcer observes level of non-compliance and chooses level of enforcement
Offender-Leadership Game

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Offender-Leadership Game

- **Time structure**
  - Stage 1: Offender chooses an observable level of non-compliance
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- Offender chooses a point on Offender’s best response curve to maximize his payoff
- The ability to commit improves offender’s payoff relative to a simultaneous game
- Level of non-compliance is lower relative to a simultaneous game
Non-Compliance not Higher in an Offender-Leadership than a Nash Game

- Suppose offender chooses $q^A$ (more non-compliance relative to Nash); enforcer replies with $p^A$
Suppose offender chooses $q^A$ (more non-compliance relative to Nash); enforcer replies with $p^A$.

Offender’s payoff is higher at B than at A ($q^B$ is a best response) and is higher at N than at A (offender’s payoff is decreasing along his reaction curve) $\Rightarrow$ Offender should choose $q^N$ rather than $q^A$.  

Non-Compliance not Higher in an Offender-Leadership than a Nash Game
Offender-Leadership Game

Offender-leadership Equilibrium
Offender-Leadership Game

- Offender maximizes gain from non-compliance minus expected sanction
Offender-Leadership Game

- Offender maximizes gain from non-compliance minus expected sanction
- Offender’s problem:

\[
\max_q \left[ (1 - p_{br}(q))G(q) - p_{br}(q)qS \right]
\]
Offender-Leadership Game

- Offender maximizes gain from non-compliance minus expected sanction
- Offender’s problem:

\[
\max_q [(1 - p_{br}(q))G(q) - p_{br}(q)qS]
\]

- First-Order Condition

\[
\begin{align*}
\text{Direct Effect} & \quad \frac{dp_{br}(q)}{dq} (G(q) + qs) = 0 \\
\text{Marginal net gain given } p & \quad \frac{(1 - p_{br}(q))G(q) - p_{br}(q)S}{dp_{br}(q)}
\end{align*}
\]

\[
\text{Strategic Effect} \\
\text{Marginal inducement loss}
\]
Stackelberg versus Nash

<table>
<thead>
<tr>
<th></th>
<th>Enforcer-Leadership</th>
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<tr>
<td>Enforcement</td>
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### Stackelberg versus Nash

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No Sanction: Enforcer has no First-Mover Advantage

• Nash and Enforcer-Leadership Equilibrium

If $S = 0$, the enforcer's best response is $q = 1$.

Enforcers cannot induce deterrence as a leader.
No Sanction: Enforcer has no First-Mover Advantage

- Nash and Enforcer-Leadership Equilibrium

\[ S = 0, \] offender’s problem reduces to \( \max_q (1 - p) G(q) \)
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- Nash and Enforcer-Leadership Equilibrium

If \( S = 0 \), offender’s problem reduces to \( \max_q (1 - p) G(q) \)

for \( p < 1 \), the offender’s best response is \( q = 1 \)
No Sanction: Enforcer has no First-Mover Advantage

- Nash and Enforcer-Leadership Equilibrium

\[ S = 0, \text{ offender's problem reduces to } \max_q (1 - p) G(q) \]
- for \( p < 1 \), the offender’s best response is \( q = 1 \)
- Enforcer’s cannot induce deterrence as a leader
No Sanction: Enforcer has a Second-Mover Advantage

- Offender-Leadership Equilibrium

\[ q^* = 1 \]

\[ q \]

\[ p^* \]

\[ 1 \]

Offender-Leadership Equilibrium
No Sanction: Enforcer has a Second-Mover Advantage

- **Offender-Leadership Equilibrium**

\[
offender' s \text{ problem as a leader is } \max_q (1 - p_{br}(q)) G(q)
\]

By lowering non-compliance relative to Nash, offender reduces the probability of detection thereby increasing his payoff.
No Sanction: Enforcer has a Second-Mover Advantage

- Offender-Leadership Equilibrium

Offender’s problem as a leader is $\max_q (1 - p_{br}(q)) G(q)$

By lowering non-compliance relative to Nash, offender reduces the probability of detection thereby increasing his payoff.
Linear Enforcement costs (Constant Marginal Enforcement Costs)

Enforcer’s marginal gain from preventing non-compliance \((qH)\) is either higher or lower than his marginal cost.
Enforcer’s marginal gain from preventing non-compliance \((qH)\) is either higher or lower than his marginal cost.

Enforcer’s best response is:

\[
p = 0 \text{ for } q < q^* \\
p = 1 \text{ for } q > q^* \\
p \in [0, 1] \text{ for } q > q^*
\]
Linear Enforcement costs: Enforcer has no Second-Mover Advantage

- Offender-leadership equilibrium

The diagram illustrates the relationship between $p^*$ and $q^*$, with $p^*$ = 0 and $q^*$ = 1 indicating the Offender-leadership Equilibrium.
Linear Enforcement costs: Enforcer has no Second-Mover Advantage

- Offender-leadership equilibrium
- When offender moves first, offender can choose a level of non-compliance infinitesimally lower than the Nash level
Linear Enforcement costs: Enforcer has no Second-Mover Advantage

- Offender-leadership equilibrium
- When offender moves first, offender can choose a level of non-compliance infinitesimally lower than the Nash level
- Enforcer’s payoff is equal to his Nash payoff since given $q^*$ enforcer’s payoff is independent of $p$
Linear Enforcement costs: Enforcer has a First-Mover Advantage

- Enforcer-leadership equilibrium

![Diagram showing Enforcer-leadership equilibrium](image-url)
Linear Enforcement costs: Enforcer has a First-Mover Advantage

- Enforcer-leadership equilibrium

- When enforcer moves first, he can induce the offender to choose a level of non-compliance lower than the Nash level
Continuous actions give rise to an offender-leadership enforcement game
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Offender enjoys both a first-mover and a second-mover advantage (relative to Nash)
Continuous actions give rise to an offender-leadership enforcement game

Offender enjoys both a first-mover and a second-mover advantage (relative to Nash)

Enforcer might prefer to be a follower than a leader if second-mover advantage greater than first-mover advantage
Continuous actions give rise to an offender-leadership enforcement game.

Offender enjoys both a first-mover and a second-mover advantage (relative to Nash).

Enforcer might prefer to be a follower than a leader if second-mover advantage greater than first-mover advantage.

Other Applications?